Incentives in the absence of verifiable results

- Tournament models – internal labour markets, promotions
- Efficiency wage models – from nutrition to involuntary unemployment

Contracts, risk-sharing and incentives

	Executives						Workers	
	PP	RPE	S	SO	SC	Daily wage	PP	Daily wage
All	32.9	3.6	3.7	3.1	14.0	1550	42.5	767
Industry								
Manufacturing	26.5	1.0	1.5	1.4	13.6	1660	38.8	810
Hi-tech manufacturing	48.9	1.5	2.1	35.1	10.2	2065	41.3	936
Construction	33.7	1.6	8.2	1.2	3.9	1325	60.1	836
Retail trade	47.4	8.5	3.3	1.7	11.1	1155	40.0	491
Wholesale trade	41.5	4.7	5.3	1.3	19.0	1731	62.1	872
Finance+priv.services	39.7	4.6	1.4	8.0	5.7	1854	60.3	940
Finance	45.2	11.2	0	30.1	42.1	2699	66.5	1226
IT	45.7	1.9	11.4	20.3	18.6	2322	73.8	1259
Business services	37.7	2.0	6.1	4.9	17.9	1805	49.3	838
Size								
11-24	29.4	2.8	3.5	1.5	12.3	1300	39.1	723
25-49	36.2	5.2	4.0	2.7	15.0	1683	48.9	807
50-99	38.5	4.2	2.4	8.1	17.4	2067	45.1	853
100-249	46.7	4.4	3.5	10.0	23.9	2537	48.8	912
250-499	49.3	7.7	15.6	21.8	16.5	3481	49.0	975
500+	66.0	8.1	9.4	23.7	28.2	4382	58.8	1093

A basic tournament model

- Basic assumptions:
  - Unverifiable production (results)
  - Unverifiable provision of effort (provided by agents)
  - Risk neutral principal
  - Risk neutral agents, fixed number N.
  - Sequence of moves:
    - The principal offers a contract (bonus and promotions)
    - The agents accept the contract or moves away.
    - A random event occurs that affects the result of the agents' effort
    - The principal promotes and pays the agents according to the contracted remuneration scheme (both promotions and bonuses are verifiable).

A basic tournament model

- Basic assumptions …..:
  - Effort is costly for the agents, $C(e)=0.5ce^2$ (conflict of interest)
  - Utility depends on remuneration (which the agent likes) and effort (which the agent dislike), $U[W-C(e)]=W-C(e)$.
  - Production for each worker independent: $y=e+\varepsilon$, where $\varepsilon \sim N(0, \sigma^2)$.
  - Principal provides a contract providing pay: $W=k$ or $W=k+b$, $b>0$,
    - k=fixed pay regardless of promotion, b=bonus following promotion,
  - and the number of promotions L ($N \geq L$). Note b and L verifiable!
- Strategy for solving the model:
  - Principal knows that the agent is utility maximizing. so step 1: find the agent's expected utility and maximize this w.r.t. effort.
  - Contingent on this info, find L and b which maximize the principal's profit, given that the agents accept the contract.

A basic tournament model

- Note: each agent know that to be promoted he or she will have to produce more than an unknown quantity $\hat{y}$.
- Since $y=e+\varepsilon$ then $\varepsilon \geq \hat{y}-e$,
- Since $\varepsilon \sim N(0, \sigma^2)$

  then $\Pr(\varepsilon < \hat{y}-e)=\Phi(\hat{y}-e) \rightarrow \Pr(\varepsilon \geq \hat{y}-e)=1- \Phi(\hat{y}-e)$
- Maximize agent's expected utility:
  - $EU=k+b[1- \Phi(\hat{y}-e)]-C(e)$
  - $\text{Max}_e\ EU \rightarrow \partial EU/\partial e =0 \rightarrow \partial\{k+b[1- \Phi(\hat{y}-e)]-C(e)\}/ \partial e=0$

    $\rightarrow -b\partial\ \Phi(\hat{y}-e)]\ \partial e-C'(e) =0 \rightarrow b\Phi'(\hat{y}-e^*)=C'(e^*)$ (and $\Phi'(\hat{y}-e^*)=\varphi(\hat{y}-e^*)$).

    (marg. expected gain (bonus)=marg.cost)

    INCENTIVE CONSTRAINT
- Optimal effort depends on b and $\hat{y}$, but we can denote this as: $e^*= e^* (b, \hat{y})$.
- How does e^* react to changes in b and $\hat{y}$?

A basic tournament model

- In optimum $b\,\varphi(\hat{y}-e^*)-C'(e^*)=0$ thus we can differentiate the FOC w.r.t. b, $\hat{y}$, and e^*. (Note: in optimum: $b\varphi'(\hat{y}-e^*)+C''>0$ (second order condition)).

- Changes in $\hat{y}$: $b\varphi'(\hat{y}-e^*)d\hat{y}-b\varphi'(\hat{y}-e^*)de^* - C''(e^*)de^* = 0$

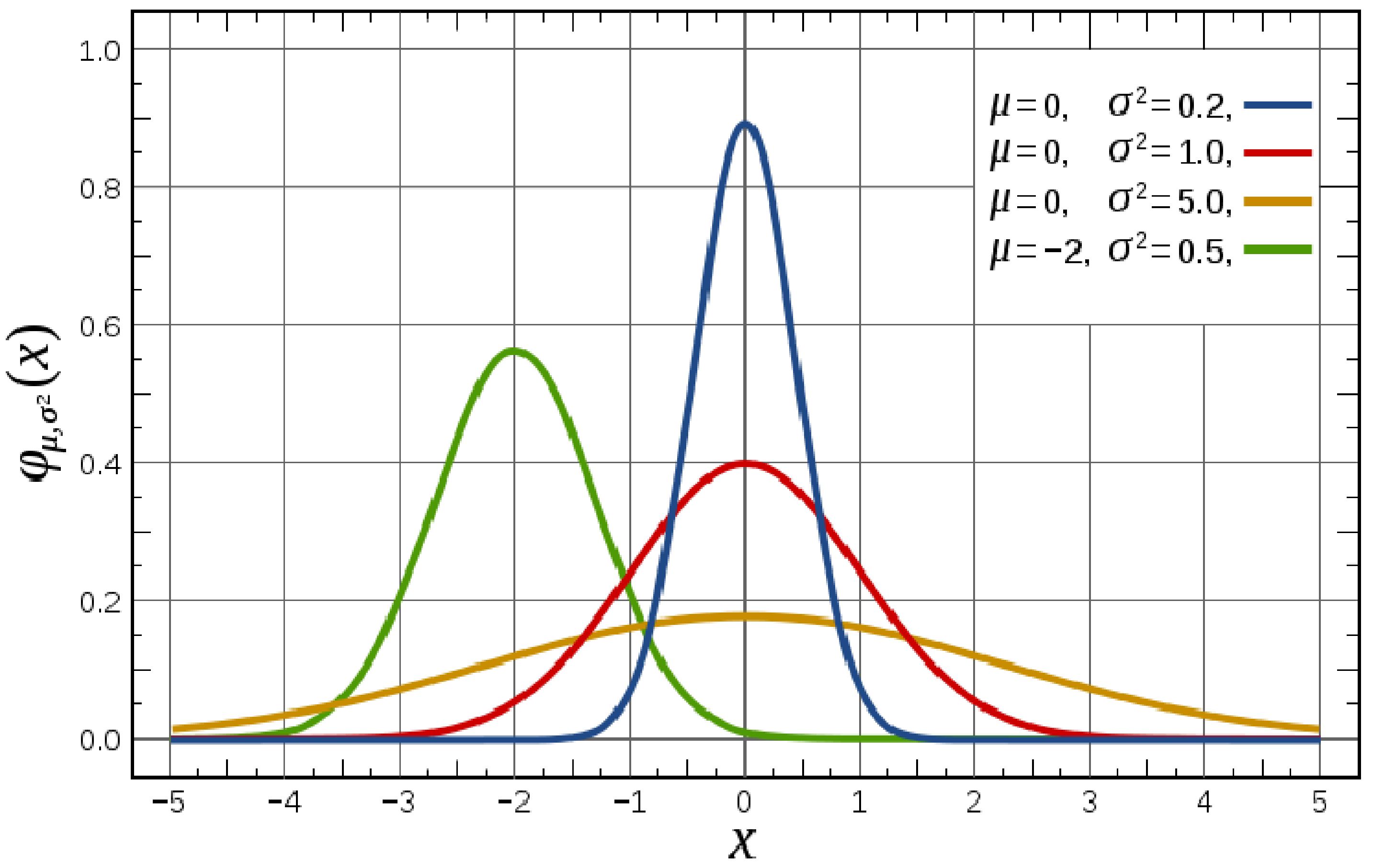
  $$\rightarrow de^*/d\hat{y}= b\varphi'(\hat{y}-e^*)/[b\varphi'(\hat{y}-e^*)+C''>0],$$

- changes in b: $\varphi(\hat{y}-e^*)db-b\varphi'(\hat{y}-e^*)de^* - C''(e^*)de^* = 0$

  $$\rightarrow de^*/db= \varphi(\hat{y}-e^*)/[b\varphi'(\hat{y}-e^*)+C''>0].$$

- What do we know about the normal distribution and density?

A basic tournament model



A basic tournament model

- Since the sign of the normal density flips at its expected value $(\hat{y}-e^*)$ we get:

- Changes in $\hat{y}$: $de^*/d\hat{y} = b\varphi'(\hat{y}-e^*)/[b\varphi'(\hat{y}-e^*)+C''>0] <=> 0$ depending on sign $(\hat{y}-e^*)$.

- Changes in b: $de^*/db = \varphi(\hat{y}-e^*)/[b\varphi'(\hat{y}-e^*)+C''>0]>0$.

- So effort increases with the size of the bonus, and if $\hat{y}-e^*$ is negative. If $\hat{y}-e^*$ is positive, then effort decreases.

A basic tournament model

- Expected profit per capita (remember each worker is identical) for the principal can be expressed:

$$E\Pi(b, \hat{y})=e^* - k - b[1- \Phi(\hat{y}-e)].$$

- Profit maximization w.r.t. b and $\hat{y}$ then gives:

$$MAX_{b,\hat{y}}\ E\Pi(b, \hat{y})=MAX_{b,\hat{y}}\{e^* - k - b[1- \Phi(\hat{y}-e)]\}$$

given 1) incentive constraint: $b\ \varphi(\hat{y}-e^*)-C'(e^*)=0$, 2) part.constraint: $EU\geq UO$

- Remember, no reason to pay more than what is needed, so EU=UO.
- Therefore $EU=k+b[1- \Phi(\hat{y}-e)]-C(e)=UO \rightarrow k+b[1- \Phi(\hat{y}-e)]=UO+C(e)$.

- Profit maximization w.r.t. b and $\hat{y}$ then gives:

$$MAX_{b,\hat{y}}\ E\Pi(b, \hat{y})=MAX_{b,\hat{y}}\{e^* - k - b[1- \Phi(\hat{y}-e)]\}=MAX_{b,\hat{y}}\{e^* - C(e^*)-UO\}$$

given 1) incentive constraint: $b\ \varphi(\hat{y}-e^*)-C'(e^*)=0$, 2) part.constraint: $EU\geq UO$

A basic tournament model

- Profit maxmization w.r.t. b and ŷ then gives:

  $MAX_{b,\hat{y}} \{e^* - C(e^*) - UO\}$

  given 1) incentive constraint: $b\,\varphi(\hat{y}-e^*)-C'(e^*)=0$, 2) part.constraint: $EU \geq UO$

- But $\{e^* - C(e^*)-UO\}$ reaches maximum at $C'(e^*)=2*0.5ce^*=1 \rightarrow e^*=1/c$.

- Thus the two constraints express two equations with two unknown, and therefore explicitly solves unique values for b and ŷ:

  $b\,\varphi(\hat{y}- 1/c)=1$, and 2) $k+b[1- \Phi(\hat{y}- 1/c)]=UO+ 1/2c$.

- Since all workers who produces $y>\hat{y}$ are promoted, the number of promoted workers are given by: $L=N[1- \Phi(\hat{y}- 1/c)]$.

  (thus whether the principal specifies L or ŷ does not matter thus no need to observe the threshold level of production!)

A basic tournament model

- What happens if the uncertainty or risks increase?

- Use Taylor-/Maclaurin-series to approximate a solution: f(x) is a complex function, then the taylor-series is $\sum^{\infty}_{n=0}(x-a)^n f^{(n)}(a)/n!$. A first order expansion is equal to $f(a)+f'(a)(x-a)$. (a=0 then Maclaurin). Note also: $\varphi(0)=1/\sigma\sqrt{2\pi}$ and $\varphi'(0)=0$, $\Phi(0)=0.5$, $\Phi'(0)= \varphi(0)=1/\sigma\sqrt{2\pi}$.

- In our case: $\varphi(\hat{y}-e^{*})\approx \varphi(0)=1/\sigma\sqrt{2\pi}$ and $\Phi(\hat{y}-e^{*})\approx 0.5+ (\hat{y}-e^{*})\varphi(0)$.
- But since $b\,\varphi(\hat{y}-e^{*})=1$ and $b[1- \Phi(\hat{y}-e^{*})]=UO+ 0.5/c - k$,

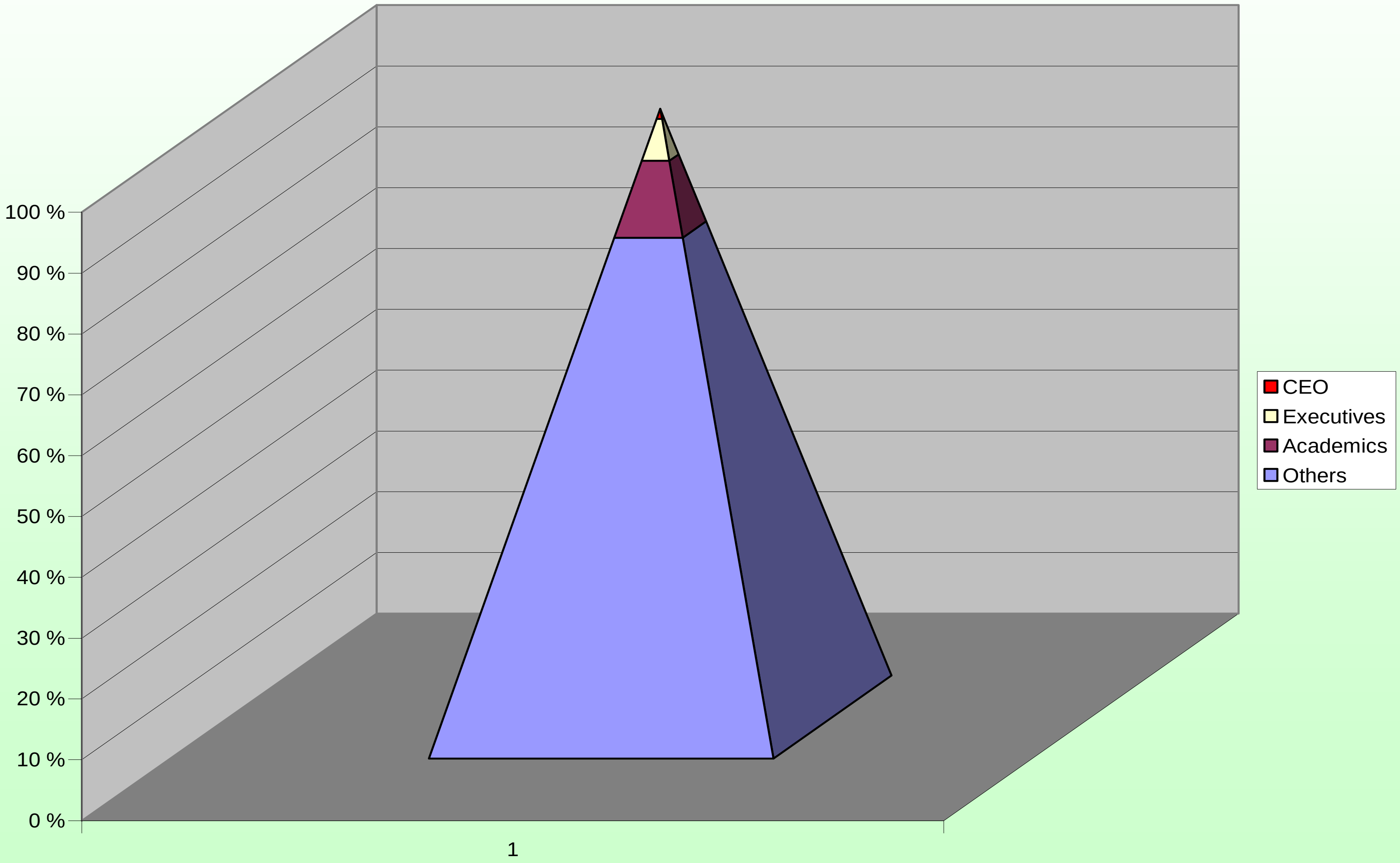
  we see directly that $b= 1/\sigma\sqrt{2\pi}$ and thus $[1- \Phi(\hat{y}-e^{*})]=[UO+ 0.5/c - k]/\sigma\sqrt{2\pi}$.

- If increased uncertainty implies higher σ, then the proportion of promotions drop (as expressed by $[1- \Phi(\hat{y}-e^{*})]$) and the bonus will have to increase.

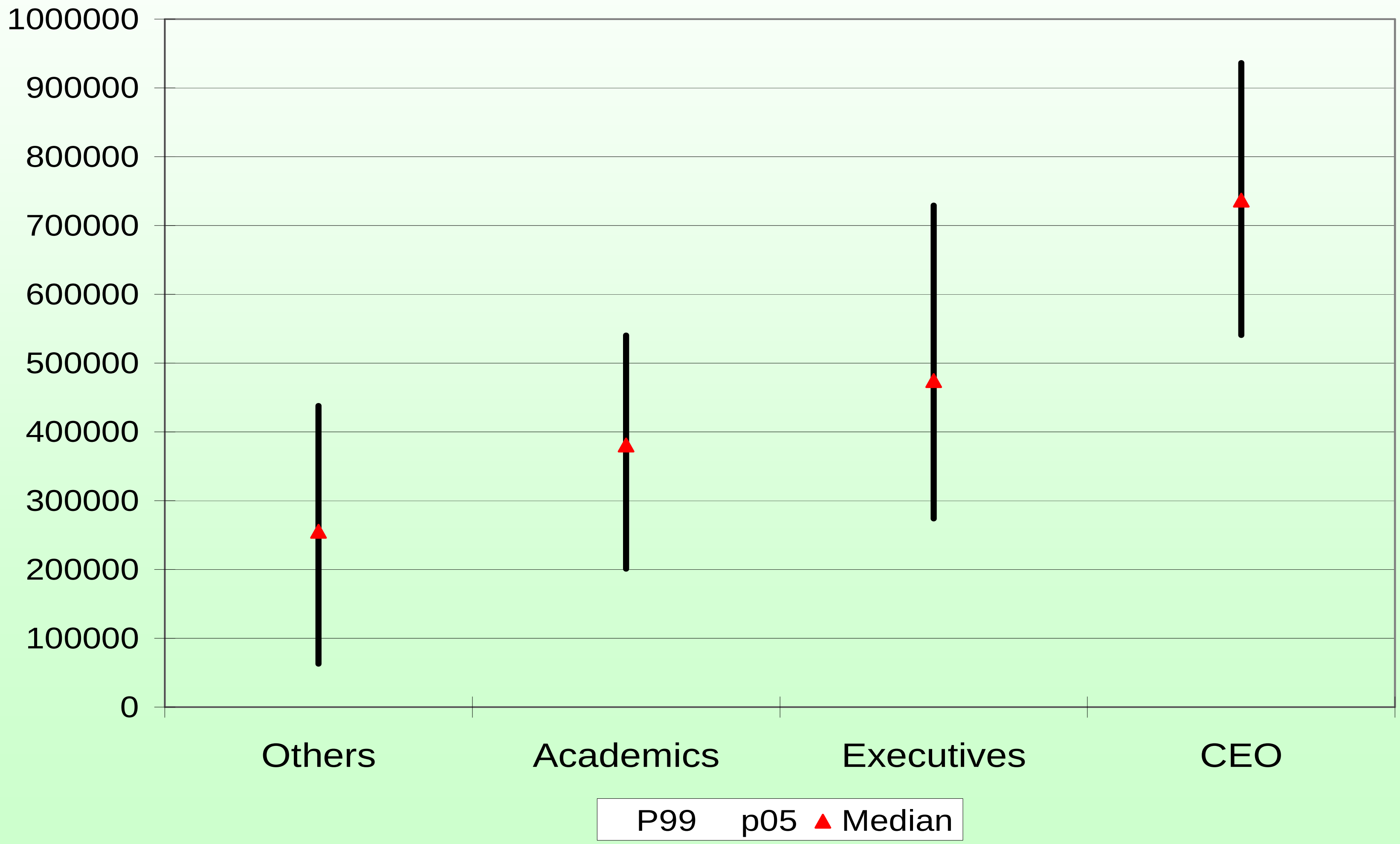
A basic tournament model

- Implications:
- In more volatile industries bonuses should be larger but promotions less frequent
- In firms with more complex organization bonuses should be larger but promotions less frequent
- Larger firms should in general pay higher wages, since more people aspire to promotions (uncertainty increases with N).

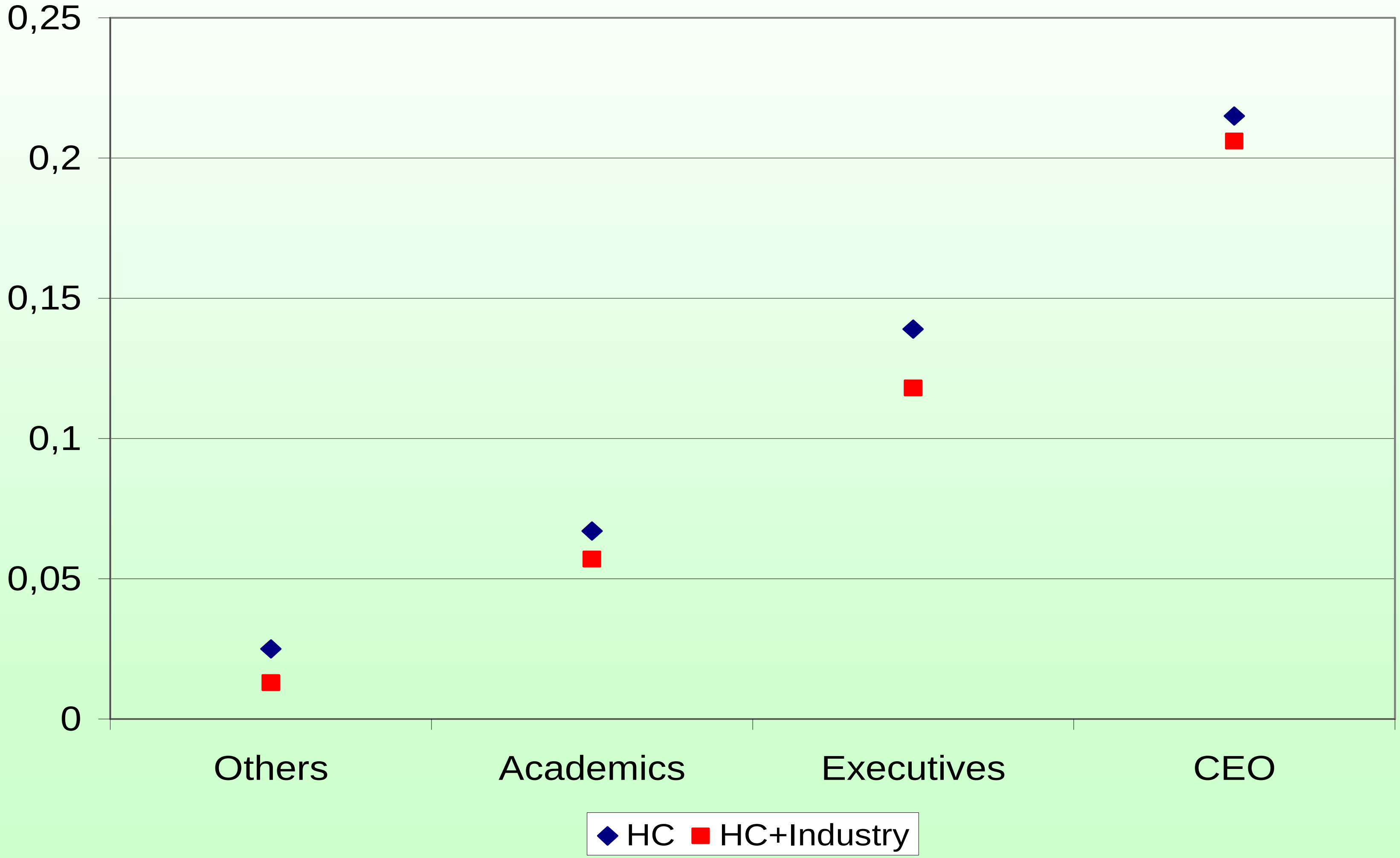
Empirical "evidence" on tournaments



Empirical "evidence" on tournaments



Empirical "evidence" on tournaments



Efficiency wage models

- Basic assumptions:
  - Unverifiable production (results)
  - Unverifiable provision of effort (provided by agents)
  - Risk neutral principal
  - Risk neutral agents, fixed number N in the economy.
  - Sequence of moves:
    - The principal offers a contract (w_t:t=0,1,2,3,4….)
    - The agents accept the contract or moves away.
    - Obligatory to provide effort e_t to produce y_t
    - The agent determines to shirk or not to shirk, effort costly C,
    - Production realised (if any)
    - The principal inspects the agent's effort imperfectly at a probability, p, less than 1.
    - If caught shirking agent fired (but keep this period wage regardless of effort provision)

Efficiency wage models

- Basic assumptions (cont….)
  - Each period an agent can lose his or her job at a exogenous probability $q>0$.
  - Each period an agent is imperfectly monitored at a exogenous probability $p<1$.
  - Let $0 \leq \delta \leq 1$ express the discount rate,
  - No search frictions.
- Principal's expected intertemporal profit from continuation after t-th period:

$$\Pi_t = y_t - w_t + \delta[(1-q)\mathrm{MAX}\{\Pi_{t+1}, \Pi^s_{t+1}\} + q\, \Pi^c_{t+1}]$$

  - Note: $q\,\Pi^c_{t+1}$ =prob. job destroyed*profit in a competitive market (after contract)
  - $(1-q)\mathrm{MAX}\{\Pi_{t+1}, \Pi^s_{t+1}\}$=prob.job not destroyed*max of next period profit or employer cheating profit,
  - $y_t - w_t$ = instantaneous profit period t

Efficiency wage models

- What does the principal get by cheating?
  - $\Pi^s_{t+1} = y_t - w_t + q\,\Pi^c_{t+1}$

- What is necessary to get principal to avoid cheating?
  - Employer's incentive constraint: $\Pi^s_t \leq \Pi_t$ for all $t \geq 0$.

- But if $\Pi^s_t \leq \Pi_t$ for all $t \geq 0$, then $\Pi^c_{t+1} \leq \Pi_{t+1}$ for all $t \geq 0$, since $\Pi_t - \Pi^s_t = y_t - w_t + \delta(1-q)[MAX\{\Pi_{t+1}, \Pi^s_{t+1}\} - \Pi^c_{t+1}]$.
  And no better alternative now exist…

  - Employer's incentive and participation constraint: $\Pi^c_{t+1} \leq \Pi_{t+1}$ for all $t \geq 0$

Efficiency wage models

- The expected intertemporal utility of an agent:

  $V_t = w_t - C + \delta[(1-q)MAX\{V_{t+1}, V^s_{t+1}\} + q\, V^c_{t+1}]$

  - Note: $q\, V^c_{t+1}$ =prob. job destroyed*utility in a competitive market (after contract)
  - $(1-q)MAX\{V_{t+1}, V^s_{t+1}\}$=prob.job not destroyed*max of next period utility after furnishing effort t or utility from shirking,
  - w_t -C = instantaneous net utility (income less effort costs) period t

- What then is the utility of shirking?
  - $V^s_{t+1} = w_t + (1-p)\delta[(1-q)MAX\{V_{t+1}, V^s_{t+1}\} + qV^c_{t+1}] + p\delta V^c_{t+1}$
  - Note: $p\delta V^c_{t+1}$=prob.caught shirking*utility next period in market
  - w_t = instantaneous net utility (income less zero effort costs) period t
  - $(1-q)MAX\{V_{t+1}, V^s_{t+1}\}$=as above

Efficiency wage models

- How does the principal avoid worker shirking?
- By ensuring that $V^s_t \leq V_t$ for all $t \geq 0$.
  - Incentive constraint
  - $V_t - V^s_t = -C + p\delta[(1-q)MAX\{V_{t+1}, V^s_{t+1}\} + q V^c_{t+1}] - p\delta V^c_{t+1}$
  - But if incentive constraint satisfied then: i) $V_t - V^s_t \geq 0$, ii) $MAX\{V_{t+1}, V^s_{t+1}\} = V_{t+1}$,
  - Therefore: $V_{t+1} - V^c_{t+1} \geq C/[pq(1-q)]$ for all $t \geq 0$.

- But this means that the value of continued employment in this contract implies a positive gain compared to what the agent get in the market….a positive rent for t=1,2,3,4……!

Efficiency wage models

- How will the final contract look like?
- We know that an optimal contract satisfies all constraints and maximizes at all t the principal's expected profits.

- It will be self-enforcing and exist if the following is satisfied:
  - i) $S_0 \geq 0$ and ii) $S_{t+1} \geq C/pq(1-q)$ for all $t \geq 0$,

    where $S_t = V_t - V^c_t + \Pi_t - \Pi^c_t$ for all $t \geq 0$. (Global surplus!)

  - The global surplus (as a difference equation) can also be expressed as a function of exogenous parameters and variables for the principal and the agent:
  - $S_t - \delta(1-q)S_{t+1} = \underbrace{y_t - C + \delta(V^c_{t+1} + \Pi^c_{t+1}) - (V^c_t + \Pi^c_t)}_{\text{exogenous}}$ for all $t \geq 0$.

    - exogenous

Efficiency wage models

- Since $S_t = V_t - V^c_t + \Pi_t - \Pi^c_t \rightarrow \Pi_t = -V_t + (V^c_t - \Pi^c_t + S_t)$
  - Since S_t determined from the difference equation contingent on exognous variables it is self exogenous, thus all variables in parenthesis are exog.
  - Thus maximization of profit Π_t is achieved by minimization of V_t.

- Since $V_{t+1} - V^c_{t+1} \geq C/[pq(1-q)]$ for all $t \geq 0$ minimization of V_{t+1} occurs at $V_{t+1} = V^c_{t+1} + C/[pq(1-q)]$. For t=0 we have $V_0 = V^c_0$. No rent first period.

- What does this mean for wages?
  - First period:

    $V_0 = w_0 - C + \delta[(1-q)MAX\{V_1, V^s_1\} + q V^c_1]$

    $= w_0 - C + \delta[(1-q)V_1 + qV^c_1]$
  - But $V_1 = V^c_1 + C/[pq(1-q)]$ so $w_0 = V^c_0 - \delta V^c_1 + C - C/p$
  - Similar technique yields: $w_t = V^c_t - \delta V^c_{t+1} + C + (C/p)[1/pq(1-q)] - C/p$

Efficiency wage models

- Assume now for simplicity (as Shapiro Stiglitz, 1984) that $V_t = V^c$ for all $t \geq 0$, thus we are assuming stationarity.
- Consequence: the same wage will be paid each period:
  - $w = (1-\delta)V^c + C + (C/p)[1/pq(1-q)] - C/p$ (efficiency wage)
- Since $V_0 - V^c = C/[pq(1-q)]$ for all $t \geq 0$ a rent arises over the whole period.
  - Getting a job is strictly superior to being unemployed, unemployed thus involuntary.
- Assume that z expresses the net gain of an unemployed person each period.
- Let $1 \geq s \geq 0$ expresses the endogenous probability of returning to work at every period (leaving unemployment).

Efficiency wage models

- The intertemporal utility of an unemployed person:
  $V^c = z + \delta[\, sV + (1\text{-}s)\, V^c \,]$

  - z=each period the instantaneous unemployment benefit
  - sV=prob getting a job*expected value from employment
  - (1-s) V^c =prob staying unemployed*exp value from unemployment

- Since we know from before that: $V - V^c = C/[p\delta(1\text{-}q)]$, we get rid of V and find: $(1-\delta)V^c = z + sC/p\delta(1\text{-}q)$

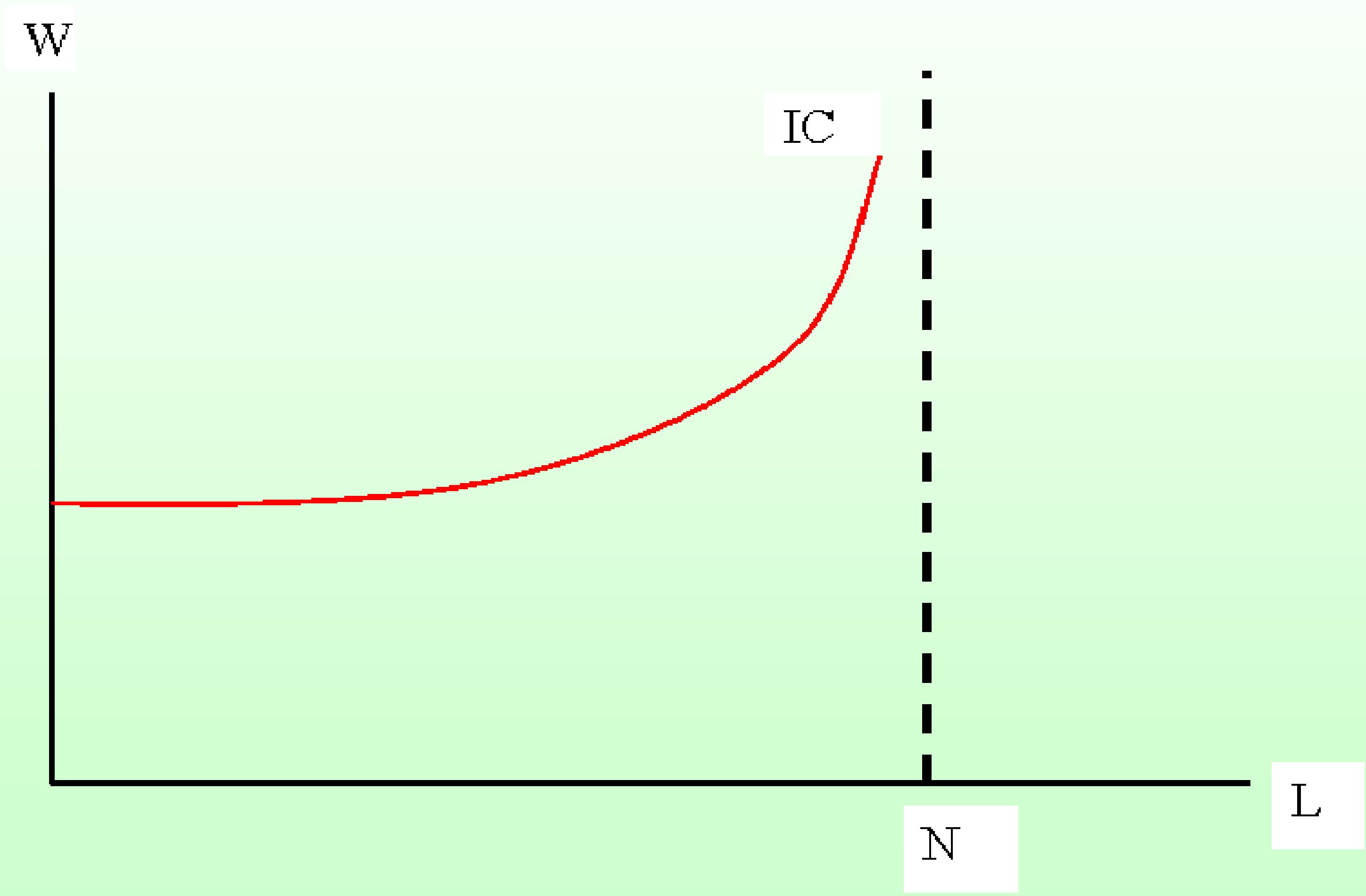
- Inserted the expression for the efficiency wage we get:
  - $w = z + C + (C/p)[\,(s+1/\delta)(1/(1\text{-}q)\,]\text{-}C/p$
  - Note the relationship between w and s (which is still endogenous and depends on L)

Efficiency wage models

- In equilibrium inflow into and outflow from unemployment has to be equal:
  - Inflow into unemployment: qL
  - Outflow from unemployment: s(N-L)
  - Since $qL=s(N-L) \rightarrow s=qL/(N-L)$

- Inserted $(s=qL/(N-L))$ into the expression for the efficiency wage we get:
  - $w = z + C + (C/p)\{[qL/(N-L)]+1/\delta)(1/(1-q)\}-C/p$
  - Incentive curve (IC), vertical asymptote at L=N.

Efficiency wage models



Efficiency wage models

- Implications:
  - Never full employment
  - The smaller the risk of lasting unemployment, the higher the pay

- For the firms, we let firms outside opportunities be equal to Π^c in the stationary state. The constant exogenous production of a worker is denoted by y. Assume s<1, i.e. more unemployed than vacant jobs.

- Expected profit from a filled job:
  - $\Pi = y - w + \delta\,[(1-q)\Pi + q\Pi^c]$, but since s<1 then $\Pi = \Pi^c$

Efficiency wage models

- Free entry equilibrium,
- Assume fixed entry costs Ck
  - Free entry equilibrium: $\Pi=\Pi^c=Ck$.
  - The smaller the risk of lasting unemployment, the higher the pay

- Thus $\Pi=y-w+\delta[(1-q)\Pi+q\Pi^c]$ implies that $\Pi=y-w^*+\delta\Pi$, which again implies $w^*=y-(1-\delta)\Pi= y-(1-\delta)Ck$ (equilibrium value of wage)

- w^*=IC-curve then solves L^* (equilibrium employment)

Efficiency wage models